TWENTY NINTH IRISH MATHEMATICAL OLYMPIAD

Saturday, 23 April 2016

First Paper

Time allowed: **Three hours.**

1. If the three-digit number ABC is divisible by 27, prove that the three-digit numbers BCA and CAB are also divisible by 27.

2. In triangle ABC we have $|AB| \neq |AC|$. The bisectors of $\angle ABC$ and $\angle ACB$ meet AC and AB at E and F, respectively, and intersect at I. If $|EI| = |FI|$ find the measure of $\angle BAC$.

3. Do there exist four polynomials $P_1(x), P_2(x), P_3(x), P_4(x)$ with real coefficients, such that the sum of any three of them always has a real root, but the sum of any two of them has no real root?

4. Let ABC be a triangle with $|AC| \neq |BC|$. Let P and Q be the intersection points of the line AB with the internal and external angle bisectors at C, so that P is between A and B. Prove that if M is any point on the circle with diameter PQ, then $\angle AMP = \angle BMP$.

5. Let $a_1, a_2, \ldots, a_m$ be positive integers, none of which is equal to 10, such that $a_1 + a_2 + \cdots + a_m = 10m$. Prove that

$$(a_1 a_2 a_3 \cdots a_m)^{1/m} \leq 3\sqrt{11}.$$